The engineer deals with ancient problems with the pen

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There are several mathematical problems that have remained unresolved since ancient times. One of them is the problem of the triangulation of the angle. Swedish-American engineer Carl Wern approached the problem in a practical way.

Several of mathematics' classical problems were formulated as early as antiquity. Some have been resolved for a long time while others have not yet received a satisfactory solution. An example of an unsolved problem is the triangulation of the angle.

The problem with the threefold division of the angle is that – as the name implies – dividing an angle into three equal parts. Dividing an angle into three equal parts is not in itself that difficult. The challenge lies in the tools that may be used – only a compass and an unmarked ruler are allowed.

The Swedish-American engineer Carl Wern, now living in Småland, last year presented his drawing solution for the three-part division of the angle. Today he is 96 years old and many of those years have been devoted to mathematics.

– Mathematics has been my best subject since I went to school, already from first grade. I got my interest in technology from my father who was an architect and builder in the USA. He had emigrated from Småland but returned to his hometown Eksjö and taught me how to draw houses with ink, Carl Wern writes in an email to Ny Teknik.

The arithmetic became a highlight of his career

His interest in mathematics and technology led Carl Wern to choose to study engineering in heating, water and sanitation (plumbing).

The calculator was awarded a gold medal at the inventors' fair in Brussels in 1966. Photo: Johan Lindqvist

– Stockholm Technical Institute was by far the best school **in my field**. The education as an engineer was then two and a half years at the day school. I took the exam in the autumn of 1947 and passed. I then went to day school for a year and read in the summer to complete, then a year to evening school. I became an engineer in the spring of 1949, he writes.

His career as an engineer led Carl Wern to the United States in 1951. There he worked for the company Robert E. Hattis Incorporated. This later led back to Sweden in 1961 and CEO position in Stockholm when the company was given project responsibility for **Mechanical and electrical design installations during the construction of Åhléns city in the early 60s. He became a registered professional mechanical enginner 1960 in Michigan, USA. Mechanical Engineering includes Water and Sewage, Sprinklers, Piping, Heating, Ventilation, Air Conditioning and Automatic controls.**
(Bold text by Carl R. Wern)

In 1966, Carl Wern developed a circular arithmetic in collaboration with his two brothers, also the engineers, George and Lars. The arithmetic was produced using the programming language Algol (abbreviation of Algorithmic Language). They demanded that the user himself be aware of the order of magnitude of the number. The decimal point needed to be moved afterwards as multiplications such as 1400×3 and 1.4×3 gave the same answer.

– It feels especially fun that my drawing aid, the calculator IWA 1638, received the recognition that it had solved the problem of putting the decimal point in the right place. It resulted in a gold medal from the Inventors' Fair in Brussels in 1966 and became a family project. Several product copies ended up on General Motors and on Boeing in the USA via my older brothers Kenneth and Roy and on Ericsson in Sweden via my younger brother Lars. Today is available the product at the Smithsonian in the United Stateswrites Carl Wern.

Together with the brothers Lars and George, Carl Wern developed a circular arithmetic. The IWA 1638 calculator (framed here) is in a copy at the Smithsonian Institution in the USA. Photo: Johan Lindqvist

The problem is unsolvable in the general case

Then back to the ancient problem of the triangulation of the angle.

In 1837, the mathematician Pierre Wantzel proved theoretically that the problem could not be solved in the general case. The tools at hand simply make a perfect three-part division of any angle possible.

Carl Wern's solution does not contradict Wantzel's proof because it is not completely accurate but offers a drawing technical solution alternative.

– By the general case is meant to cope with optional angles. Then the division of three is an insoluble mathematical problem if it is to be an exact solution. But I wanted to report what can be achieved technically as perfectly as possible. As a comparison, no maths gave exact answers, but they were good enough. My goal with the three divisions was to present a drawing technical solution where the accuracy is fine, as are the answers to the math sticks. Previously, no one had reported that this was technically possible, he writes.

Carl Wern's solution is thus a practical solution that divides any angle into three almost identical parts.

Step 1: Divide an angle into two equal parts

To divide an angle into three parts, you must start by dividing the angle in two with only one unmarked ruler and a compass. This is easily done as below.

An angle is easily divided into two with just a compass and unmarked ruler. Photo: Bill Burrau

Divide the angle ABC in two by drawing a circle 1 with the center of the compass in the tip B. The circle 1 intersects the section AB and BC at each point. Then draw two new circles 2 and 3 with these intersections as the center.

The two new circles then intersect at two points. Draw a straight line between these two points and the tip B of the angle – you have now divided the angle into two equal parts.

Step 2: How to divide the angle into three

Start by drawing an angle ABC. In the accompanying image, the angle is approximately 115 °.
Divide the angle in two in a known manner and draw a straight line L through the angle.

3. After this, draw circles 1 and 2, two circles with the same radius around points B and H. Circle 1 intersects L at points H and K while circle 2 intersects L at points D and B.

4. Then draw arcs 3 and 4 from points D and B.

5. Then draw the straight line 5 through the point H and the point of intersection between arcs 3 and 4.

6. Draw a straight line 6 between the intersection of the circle 1 and the section CB to the point D. Line 6 intersects the line 5 at the point F.

7. Draw circle 7 from H with radius HF.

8. Then draw an equal circle 8 around the point K. The circle 8 and the circle 1 intersect at two points and from these the sections RB and WB can be constructed.

9. Draw the sections RB and WB.

10. The angle V ° has now been divided into three parts.

Carl Wern’s solution means that with fine precision you can divide an angle with just a compass and an unmarked ruler. Photo: Bill Burrau

One more detailed solution with the possibility of doubling the accuracy presented on Carl Wern’s website. There is also a method that increases the accuracy for angles above 120 degrees. If you do not have access to compasses and unmarked ruler, you can try to triple different angles using an animation made by the German mathematician Reinhard Atzbach.

Professor: Ancient problems can lead to new mathematics

Mats Boij is a professor of mathematics at KTH and is well aware of the problem with the threefold division of the angle. For him, however, the mathematical interest does not lie in the problem itself.– What is interesting about this problem from a mathematical point of view is verything that it has led to when trying to solve it. The mathematics that was developed to be ble to prove that it is not possible to solve with the conditions of the problem formulation is in itself very interesting. The problem itself has given rise to mathematics that is more interesting than the problem itself, says Mats Boij.

Mats Boij, professor of mathematics at KTH. Photo: Julia Sixtensson

Other mathematical problems have also led to useful and interesting mathematics.

– Another very well-known example is Fermat's large theorem which was formulated in the 17th century. There were many who worked with that problem for hundreds of years and in the end Andrew Wiles managed to formulate a proof of the 1990s. Although the problem was the incentive for the work, the things Wiles has in his evidence are very general and can be used in several different contexts, says Mats Boij.

Overall, Mats Boij thinks that problems like these may be worth looking at and trying to solve, but that one should not have too high hopes.

– It can be stimulating to have a problem like a carrot, something to work towards. But it may not always be the problem that is interesting, but what comes up along the way. But at the same time you should have respect for the fact that a lot has often been done and that you may not come up with something revolutionary, says Mats Boij.

Carl Wern's solution can be done in two ways, here Carl shows the solution that provides increased accuracy. Photo: Johan Lindqvist

A challenge for readers

When Carl Wern had created his solution, he gave it to acquaintances interested in mathematics, including Åke Tegengren, also a former engineer, and Reinhard Atzbach, a mathematician in Germany.

Tegengren saw Wern's solution and after a while came to the conclusion that it allows three divisions of the angle even if you do not adjust the compass.

How to do this, we leave to your readers.

Clue: the triad is displayed there outside the original angle ABC. Please note that it is then not possible to increase the accuracy in a final technical step as shown by Wern on his website.

More unsolved problems from antiquity

Another unsolved problem is the Delisk problem, which is about cubes. It involves constructing the edge of the cube whose volume is twice as large as the volume of a given cube. Another problem is the square of the circle. Here, a square must be constructed that has the same area as a given circle.
As with the problem of triangulation, the challenge lies in the limited tools. It is important to solve the problem only with a compass and an unmarked ruler.